MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2022/2023 (TERM ID 2210 & TERM ID 2215)

EEM2036 – ENGINEERING MATHEMATICS III

(ME, RE, TE)

13 FEBRUARY 2023
2.30 p.m. – 4.30 p.m.
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This exam paper consists of **7 printed pages** (including cover page and formula sheets) with **four questions** only.

2. Answer **ALL** questions. All necessary working steps must be shown.

3. Write all your answers in the answer booklet provided.

Please answer ALL questions and show the necessary working. Total mark is equal to 100.

Question 1 (25 marks)

a) Find the eigenvalues and the corresponding eigenvectors for the system of differential equation below. (Do not solve the question)

$$x_1' = x_1 - x_2$$
$$x_2' = -x_1 + 2x_2 - x_3$$
$$x_3' = -x_2 + x_3$$

[12 marks]

b) Given the following differential equation:

$$\frac{dx_1}{dt} = 2x_1 - x_2$$
$$\frac{dx_2}{dt} = -x_1 + 2x_2$$

i) Find the general solution to the differential equation above given that $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ are eigenvectors for the corresponding eigenvalues 1 and 3 for matrix $A = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$. [8 marks]

ii) Hence, solve the differential equation if $x_1(0) = 0$ and $x_2(0) = 1$. [5 marks]

Question 2 (25 marks)

a) Given $\int_{0.1}^{1.3} xe^{-x}\,dx$

i) Evaluate the integration above by using the Composite Trapezoidal rule with 3 intervals. [6 marks]

ii) Find the error bound. [4 marks]

iii) Find the absolute error. [4 marks]

iv) Is the absolute error within the error bound? Justify your answer. [1 mark]

Continued...

b) Consider the following system of linear equations:

$$7x_1 - 2x_2 + 3x_3 = 5$$
$$x_1 + 6x_2 - 4x_3 = 10$$
$$x_1 - x_2 - 3x_3 = 6$$

Set up the Jacobi iterative equations and perform two iterations starting with the initial approximation (0,0,0). [10 marks]

Question 3 (25 marks)

a) Let R be the region in the first quadrant bounded by the parabolas $x = y^2$, $x = y^2 - 4$, $x = 9 - y^2$ and $x = 16 - y^2$. Evaluate $\iint_R y^2\, dA$ by using the change of variables $u = x - y^2$ and $v = x + y^2$. [13 marks]

b) Evaluate $\iint_R (x+y)\, dA$ where $R = \{(x,y)/1 \le x^2 + y^2 \le 4;\ x \le 0\}$ by using polar coordinates. [6 marks]

c) Use cylindrical coordinates to find the volume of the solid that is bounded above by the hemisphere $z = \sqrt{4 - x^2 - y^2}$, below by the xy-plane and laterally by the cylinder $x^2 + y^2 = 1$. [6 marks]

Continued...

Question 4 (25 marks)

a) Consider the vector field $\vec{F}(x,y,z) = \left(ye^x + x\right)\hat{i} + \left(e^x + y\right)\hat{j} + \left(z+1\right)\hat{k}$.

i) Show that F is a conservative vector field. [3 marks]

ii) Find a potential function for $\vec{F}$. [9 marks]

iii) Hence, evaluate $\int_C F.dr$ where C is a simple curve from $A(0,2,0)$ to $B(0,0,1)$. [3 marks]

b) Use the Stokes' Theorem to evaluate $\int_C \vec{F}.dr$ where $\vec{F}(x,y,z) = y^2\,\hat{i} + x\,\hat{j} + z^2\,\hat{k}$, where C is the curve of intersection of the solid $z = x^2 + y^2$ and $z = 1$, oriented counterclockwise as viewed from above. [10 marks]

Continued...

APPENDIX

TABLE OF FORMULAS

1. The *n*th Lagrange interpolating polynomial (LIP)

$$f(x) \approx P_n(x) = \sum_{i=0}^{n} L_i(x) f(x_i)$$

with

$$L_k(x) = \prod_{\substack{i=0 \\ i \neq k}}^{n} \frac{(x - x_i)}{(x_k - x_i)}.$$

2. Newton's divided-difference interpolating polynomial (NDDIP)

$$P_n(x) = f[x_0] + \sum_{k=1}^{n} f[x_0, x_1, ..., x_k](x - x_0) \cdots (x - x_{k-1}) .$$

3. The error in interpolating polynomial.

$$f(x) - P_n(x) = \frac{(x - x_0)(x - x_1)...(x - x_n)}{(n+1)!} f^{(n+1)}(c_x)$$

for each $x \in [x_0, x_n]$, a number $c_x \in (x_0, x_n)$ exists.

4. Newton's forward-difference formula

$$P_n(x) = f[x_0] + \sum_{k=1}^{n} \binom{s}{k} \Delta^k f(x_0) .$$

5. Newton's backward-difference formula

$$P_n(x) = f[x_n] + \sum_{k=1}^{n} (-1)^k \binom{-s}{k} \nabla^k f(x_n) .$$

6. Forward difference formula

$$f'(x) \approx \frac{f(x+h) - f(x)}{h} .$$

Backward difference formula

$$f'(x) \approx \frac{f(x) - f(x-h)}{h} .$$

The error term for both forward and backward difference formula is

$$\left| \frac{h}{2} f''(c_x) \right| .$$

Continued...

7. Central difference formula

$$f'(x) \approx \frac{f(x+h) - f(x-h)}{2h}$$

with the error term

$$\left| \frac{h^2}{6} f^{(3)}(c_x) \right|.$$

8. Trapezoidal rule

$$\int_a^b f(x)dx \approx \frac{h}{2}[f(a) + f(b)]$$

for some ξ in (a, b) and $h = b - a$, with the error term is $\left| \frac{h^3 f''(\xi)}{12} \right|$.

9. Composite Trapezoidal rule

$$\int_a^b f(x)dx \approx \frac{h}{2}\left[f(a) + f(b) + 2\sum_{j=1}^{n-1} f(x_j) \right]$$

for some ξ in (a, b) and $h = \frac{b-a}{n}$, with the error term is $\left| \frac{(b-a)h^2 f''(\xi)}{12} \right|$.

10. Simpson's rule

$$\int_a^b f(x)dx \approx \frac{h}{3}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b) \right]$$

for some ξ in (a, b) and $h = \frac{b-a}{2}$, with the error term $\left| \frac{h^5}{90} f^{(4)}(\xi) \right|$.

11. Composite Simpson's rule

$$\int_a^b f(x)dx \approx \frac{h}{3}\left[f(a) + 2\sum_{j=1}^{\left(\frac{n}{2}\right)-1} f(x_{2j}) + 4\sum_{j=1}^{\frac{n}{2}} f(x_{2j-1}) + f(b) \right]$$

for some ξ in (a, b) and $h = \frac{b-a}{n}$, with the error term $\left| \frac{(b-a)h^4}{180} f^{(4)}(\xi) \right|$.

12. Newton-Raphson's method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \qquad n = 0,1,2,...$$

Continued ...

13. Euler's method

$$y_{i+1} = y_i + hf(x_i, y_i)$$

with local error $\frac{h^2}{2}Y''(\xi_i)$ for some ξ_i in (x_i, x_{i+1}).

14. Runge Kutta method of order two (Improved Euler method)

$$y_{i+1} = y_i + \tfrac{1}{2}(k_1 + k_2)$$
$$k_1 = hf(x_i, y_i)$$
$$k_2 = hf(x_i + h, y_i + k_1)$$

15. Runge Kutta method of order four

$$k_1 = hf(x_i, y_i),$$
$$k_2 = hf(x_i + \tfrac{1}{2}h, y_i + \tfrac{1}{2}k_1),$$
$$k_3 = hf(x_i + \tfrac{1}{2}h, y_i + \tfrac{1}{2}k_2),$$
$$k_4 = hf(x_{i+1}, y_i + k_3),$$
$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4).$$

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